SCATTERING BY DIELECTRIC OBSTACLES INSIDE GUIDING STRUCTURES

Abbas S. Omar and Klaus Schünemann

Technische Universität Hamburg-Harburg, Arbeitsbereich
Hochfrequenztechnik, Postfach 90 14 03, D-2100 Hamburg 90, W.-Germany

Summary

Green's function method is applied to the problem of dielectric resonators inside conducting cavities or waveguides. Systematic solutions are derived and applied to the calculation of resonant frequencies, radiation quality factors, and coupling coefficients.

INTRODUCTION

In principle, there is no essential difference between the problems of scattering by dielectric obstacles in free space and in guiding structures. The only difference is the nature of Green's function which is a scalar function in free space but a complicated dyadic one in guiding structures. The knowledge of the normal waveguide modes systemizes, however, the problem of finding the dyadic Green's function. It is the scope of this paper to construct such solutions and to apply techniques used to study scattering in free space, /1/ - /4/, to scattering inside waveguides.

The method is not only limited to dielectric obstacles but can also be applied to conducting bodies. Applications of dielectric obstacles include dielectric resonators as most important ones. Resonant frequencies, radiation quality factors, and various coupling coefficients must be computed. We will show how this can be done by the present method.

BASIC FORMULATION

Referring to Fig. 1, the total e.m. field $\underline{E}$, $\underline{H}$ inside the waveguide can be divided into two parts: the incident field $\underline{E}^i$, $\underline{H}^i$ and the scattered field $\underline{E}^s$, $\underline{H}^s$ with $\underline{E}^s=\underline{E}-\underline{E}^i$, $\underline{H}^s=\underline{H}-\underline{H}^i$. The scattered field is known to be excited by the polarization current

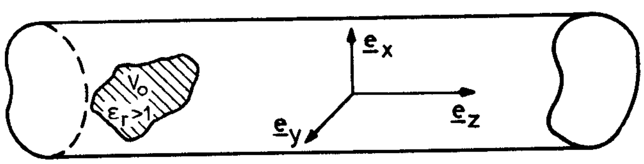

Fig. 1 Dielectric body inside a waveguide

$$\underline{J}=j\omega\varepsilon_o(\varepsilon_r-1)\underline{E}, \qquad (1)$$

where $\underline{E}$ has to be taken inside the dielectric body. This current can be shown to possess some important features. Because of (1), $\underline{J}$ must not show any divergence. Hence

$$\underline{\nabla}\cdot\underline{J}=0. \qquad (2)$$

In addition, it should satisfy the homogeneous Helmholtz equation, i.e.

$$\nabla^2\underline{J}+\varepsilon_r k_o^2\underline{J}=0. \qquad (3)$$

Boundary conditions need not to be specified, because $\underline{J}$ will be determined from an integral equation.

The scattered field, which is excited by a current source, is given by /5/

$$\underline{E}_t^s=-L_t(\underline{J}_t,J_z),\quad E_z^s=-L_z(\underline{J}_t,J_z) \qquad (4)$$

where subscript "t" denotes the transverse and "z" the longitudinal coordinate. The operators on the right hand side of (4) are defined in /5/, /6/. They contain the normal modes of the embedding waveguide.

Writing the scattered field has

$$\underline{E}^s=(j\omega\varepsilon_o(\varepsilon_r-1))^{-1}\underline{J}-\underline{E}^i \qquad (5)$$

we find two coupled integral equations for determining the current source:

$$\underline{J}_t+j\omega\varepsilon_o(\varepsilon_r-1)L_t(\underline{J}_t,J_z)=j\omega\varepsilon_o(\varepsilon_r-1)\underline{E}_t^i \qquad (6a)$$

$$J_z+j\omega\varepsilon_o(\varepsilon_r-1)L_z(\underline{J}_t,J_z)=j\omega\varepsilon_o(\varepsilon_r-1)E_z^i \qquad (6b)$$

The most suitable way for solving these coupled integral equations is to apply the moment method by expanding the unknown current source as

$$\underline{J}_t=\sum_m A_m\underline{U}_{tm},\quad J_z=\sum_m A_m U_{zm} \qquad (7)$$

where $\underline{U}_m$ are known basis functions and A_m are unknown coefficients /7/.

Relations (2) and (3) which must be fulfilled by $\underline{J}$ establish a guideline for choosing $\underline{U}_m$. These functions can be chosen as source-free electric fields inside the dielectric body with half of them satisfying electric and the other half satisfying magnetic wall boundary conditions at the dielectric surface. Thus the set of functions ($\underline{U}_m$) will represent a complete solution.

CYLINDRICAL DIELECTRIC OBSTACLE WITH ARBITRARY CROSS SECTION

The determination of the unknown current source $\underline{J}$ is rather systematic if the dielectric body is cylindrical with its axis in parallel to the waveguide axis (Fig. 2). Its cross section is arbitrary. Then (7) can be written as

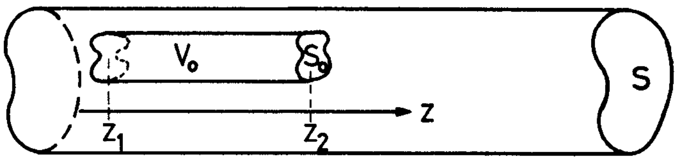

Fig. 2 Cylindrical dielectric body inside a waveguide

$$\underline{J}_t=\sum_m \frac{d}{dz} G_m(z)\underline{U}_m(\underline{r}),\quad J_z=-\sum_m G_m(z)\underline{\nabla}_t\underline{U}_m(\underline{r}). \qquad (8)$$

$\underline{r}$ means transverse coordinate vector. Furthermore

$$\nabla_t^2\underline{U}_m+k_m^2\underline{U}_m=0,\quad G_m=A_m\exp(\gamma_m z)+B_m\exp(-\gamma_m z),$$
$$\gamma_m^2=k_m^2-\varepsilon_r k_o^2. \qquad (9)$$

Substituting $\underline{J}$ in the coupled integral equations (6), it turns out that the second (6b) linearly depends on the first (6a). Hence (6a) sufficiently describes the problem.

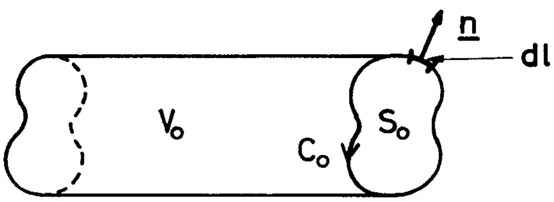

Fig. 3 The cylindrical dielectric body

Referring to Fig. 3, the basis functions $\underline{U}_m$ can be either of TE or of TM type and they can satisfy either electric or magnetic wall boundary conditions on the surface. Thus one can construct 4 sets of basis functions. Following the standard procedure of the moment method (i.e. testing the integral equation by these 4 sets), we finally get a matrix equation of form

$$\underline{\underline{M}}\ \underline{X} = \underline{Y}. \qquad (10)$$

The column vector $\underline{X}$ contains the unknown expansion coefficients A_m and B_m, while the elements of the column vector $\underline{Y}$ are given in terms of the incident field $\underline{E}^i$.

Equating det(M) to zero determines the resonant frequencies f_i of the system and the corresponding radiation quality factors Q_i. The corresponding resonant vectors $\underline{X}_i$ are obtained from the undetermined system

$$\underline{\underline{M}}(f_i,Q_i)\underline{X}_i=0. \qquad (11)$$

The coupling coefficients between different resonant modes and a certain incident field can be determined by expanding vector $\underline{X}$ in terms of the resonant vector $\underline{X}_i$ and solving (10) for the expansion coefficients which are identical to the coupling coefficients.

NUMERICAL RESULTS

Two problems have been regarded in order to check the validity of our approach.

1. A circular cylindrical dielectric resonator is symmetrically mounted inside a circular cylindrical metal cavity as sketched in Fig. 4. The resonant frequency of the lowest order resonant mode is tabulated in Table 1 for different dimensions. The agreement to results taken from /8/ is satisfactory.

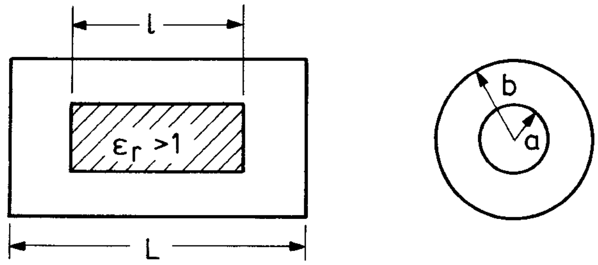

Fig. 4 Circular cylindrical dielectric body inside a circular metal cavity

2. Now a dielectric pillbox is located inside a rectangular waveguide as shown in Fig. 5. The radiation quality factor Q_r is tabulated in Table 2 for two different positions of the dielectric resonator. The agreement to experimental results and those taken from /9/ is again satisfactory.

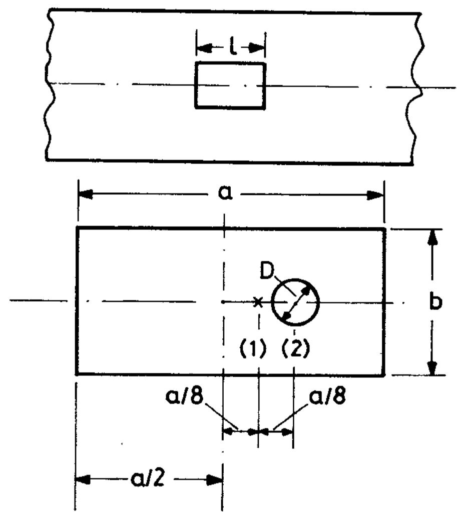

Fig. 5 Circular cylindrical dielectric resonator inside a rectangular waveguide. The dimensions are as in /9/.

Acknowledgement: The authors gratefully acknowledge the Deutsche Forschungsgemeinschaft for financial support.

REFERENCES

/1/ R.F. Harrington, J.R. Mautz, Y. Chang, "Characteristic modes for dielectric and magnetic bodies," IEEE Trans., vol. AP-20, pp. 194-198, 1972.

/2/ Y. Chang, R.F. Harrington,"A surface formulation for characteristic modes of material bodies," IEEE Trans., vol. AP-25, pp. 789-795, 1977.

/3/ J.R. Mautz, R.F. Harrington, "Electromagnetic scattering from a homogeneous material body of revolution," AEÜ, vol. 33, pp. 71-80, 1979.

/4/ A.G. Papayiannakis, E.E. Kriezis, "Scattering from a dielectric cylinder of finite length," IEEE Trans., vol. AP-31, pp. 725-731, 1983.

/5/ R.E. Collin, "Field theory of guided waves," McGraw Hill, New York 1960.

/6/ P.H. Pathak, "On the eigenfunction expansion of electromagnetic dyadic Green's functions," IEEE Trans., vol. AP-31, pp. 837-846, 1983.

/7/ R.F. Harrington, "Field computation by moment methods," Macmillan, New York 1964.

/8/ K.A. Zaki, A.E. Atia, "Resonant frequencies of dielectric loaded waveguide cavities," Proc. MTT-S, pp. 421-423, Boston 1983.

/9/ J. van Bladel, "Dielectric resonator in a waveguide above cutoff," AEÜ, vol. 32, pp. 465-472, 1978.

a(mm)	b(mm)	l(mm)	L(mm)	ε_r	This method (GHz)	Ref. /8/ (GHz)	Measured (GHz)
10.00	12.7	8.00	15.24	37.60	3.372	3.368	3.371
8.66	12.7	8.10	21.10	37.25	3.932	3.928	3.930
8.00	12.7	6.91	14.22	37.60	4.193	4.196	4.192
6.80	12.7	5.59	12.19	38.20	5.003	4.994	5.001

Table 1: Resonant frequency of the resonance system shown in Fig. 4

position \ Q_r	This method	Ref. /9/	Experimental
(1)	740	762	734
(2)	201	218.5	197

Table 2: Radiation quality factor of the resonance system shown in Fig. 5